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Cutting, by 'pressing and slicing,' of thin floppy slices of materials illustrated by experiments on cheddar cheese and salami

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Why it is easier to cut with even the sharpest knife when 'pressing down and sliding' than when merely 'pressing down alone' is explained. A variety of cases of cutting where the blade and workpiece have different relative motions is analysed and it is shown that the greater the 'slice/push ratio' ξ given by (blade speed parallel to the cutting edge/blade speed perpendicular to the cutting edge), the lower the cutting forces. However, friction limits the reductions attainable at the highest ξ. The analysis is applied to the geometry of a wheel cutting device (delicatessan slicer) and experiments with a cheddar cheese and a salami using such an instrumented device confirm the general predictions.

1. Introduction

This paper will answer the question of why it is relatively difficult, even with a sharp knife, to cut when simply 'pressing down' but much easier to cut as soon as some sideways sawing or slicing action is introduced. Before arriving at that point we shall review the mechanics of cutting.

All types of materials having widely different mechanical properties are cut in order to separate one part from another. Sometimes it is the portion cut off that is important: sometimes what is removed is waste and what is left is important. Sometimes both are important. At one extreme there is the cutting of paper with a razor blade, say, and at another there is the machining of ductile metals. In the former case, the sheet of paper remains globally elastic and the cut pieces may be recombined to form the original size piece of paper. In the latter case, the waste material in the chip is highly distorted and plastically deformed, and the original workpiece cannot be regained from the cut pieces. Further examples of the sort of globally-elastic cutting considered in this paper are to found in the slicing of meat by the butcher, the microtoming of thin sections in biology, lawn mowing, hair cutting, the cutting of fabrics by the dressmaker, surgery and so on. These cases are characterised by the offcuts being elastically very floppy (i.e., have negligible bending resistance and are not permanently deformed). That is not a prerequisite to enable the severed components to be refitted, however: offcuts may have elastic bending stiffness as in the splitting of slate, or cleavage of diamond, or chopping of wood along the grain with an axe, in all of which cases the severed pieces may be refitted.

For present purposes, we shall confine ourselves to the production of floppy offcuts that are not permanently deformed and have negligible bending resistance. However, as discussed later, the conclusions are general and apply to the cutting of the most ductile materials where there is severe permanent distortion of the offcut.

What forces and work are required to perform this type of cutting? We have for an increment of displacement $\mathrm{d}v$ in the direction of the cutting force V

$$V\,\mathrm{d}v = Rw\,\mathrm{d}a + \mathrm{d(friction)} \qquad (1)$$

where a is the cut length, w is width or thickness of the material being cut (width for microtoming; thickness for orthogonal guillotining) and R is fracture toughness. Fig. 1 shows that, so long as the cut does not run ahead of the tip of the blade at a velocity different from that of the blade (a question of crack stability), the new surface length and the blade displacement are equal ($\mathrm{d}a = \mathrm{d}v$) and provide a coupling between external and internal work increments. When friction is very small, it follows that

$$V = Rw \qquad (2)$$

The cutting force is then determined solely by the fracture toughness parameter and Equation 2 describes low-friction cutting where the offcut is incapable of storing elastic strain energy and also is not permanently deformed. For example, beef might have $R \sim 400$ J/m^2

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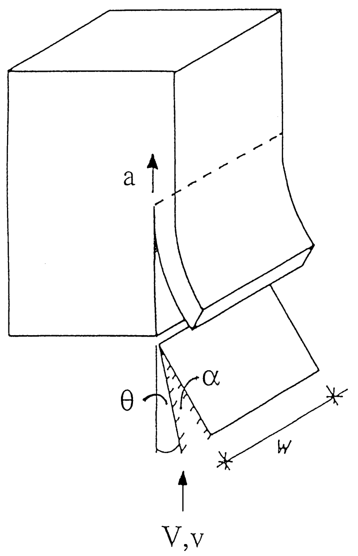

Figure 1 Idealised steady-state cutting where all the work is fracture toughness work (no friction, floppy undistorted offcut). The incremental movement of the cut surface front da keeps pace with the movement of the tool dv.$R = V/w$ where V is the cutting force and w is width of cut or thickness of material, depending upon whether a 'surface' or 'edge' cut is being taken.

so a butcher cutting a piece 150 mm wide would exert a downwards force of $400 \times 150 \times 10^{-3} = 60$ N. The well-known relation (2) is often employed the other way around, to determine the fracture toughness R of materials from measured forces in, for example, microtoming and guillotining. The same type of relation is found for 'free tearing' or ripping of floppy materials, as in the so-called 'trousers tear test' to determine toughness.

Notice that neither the cutting blade sharpness, nor its tip angle (α) nor its relief angle to the cut surface (θ) enters the picture: these aspects will be discussed later.

Friction is often important in cutting. Metalcutting investigators have employed a variety of representations for friction between cutting tool and workpiece (e.g., Childs *et al.* [1]). Since floppy offcuts (in theory) have no contact stress against the blade, so that Coulomb friction is inappropriate, we represent friction by some constant stress τ_f acting over a contact length L between cutting blade and material on each side of the blade [2]. In metalcutting τ_f is often written $\tau_f = m\tau_y$ where τ_y is the shear yield stress and where $0 < m < 1$; for $m = 1$, 'sticking friction' takes place where the workpiece finds it easier to slip within itself rather than at the cutting tool/workpiece interface and material transfer occurs, i.e., bits of the cut material stick to the blade. The increment of friction work d(friction) becomes ($2Lw\tau_f\, dv$). [The factor 2 is employed when there is contact between both sides of the blade and offcut; there are, of course, situations where the blade is in contact on only one side, and the analysis should then drop the factor 2.] Hence from Equation 1

$$\begin{aligned} V &= w(R + 2L\tau_f) \\ &= Rw[1 + \{2L\tau_f/R\}] \\ &= Rw[1 + M] \end{aligned} \tag{3}$$

where $M = [2L\tau_f/R]$. The effect of friction is to give an apparent enhancement of the toughness.

When the offcut is no longer floppy, is elastically stiff and stores bending strain energy (as in splitting a lath of wood, say), Equation 1 can be extended to include incremental bending work. Now the cutting blade angle and relief angle enter the picture since the greater the angle the greater the curvature of the offcut, and the greater the bending strain energy (cf. [3]). It may be shown [4] that there is an optimum blade angle for cutting where V is least. The reason for the existence of a least cutting force is because of the competition between friction and bending work. At small angles the friction contribution is great because the contact between blade and offcut is great, but at large angles the friction is small. Vice versa for the bending work which is small at small angles (small curvature) but large at large angles. The explanation for optimum die angles in wire drawing is the same, except that wire drawing strains are irreversible [5].

The foregoing explains the skill of the microtomist in setting up the instrument to produce 'best' (least damaged) sections. The operator is, in fact, finding the blade setting for minimum force. With hindsight, it is not surprising that smallest microtoming force damages the cut sections least (fewest internal tears between different components of micro/macrostructure etc.). Furthermore the remaining surface on the bulk piece being cut, which forms the top part of the next microtomed slice, is also least damaged which is advantageous. It is possible to instrument a microtome for forces and fit a control system which automatically finds the optimum blade orientation [4, 6, 7].

Equations 1–3 are directly applicable to cutting where the blade cuts the whole width or thickness at one go. It requires modification when (i) the blade cuts at a (fixed) angle to the direction of motion of the workpiece and (ii) when it cuts at a continuously changing angle to the direction of motion of the workpiece. These aspects are beyond the scope of the present paper and will be published elsewhere.

2. Sideways moving orthogonal blade, floppy offcut

Consider Fig. 2 which shows schematically a knife cutting a block of material of width w. The knife moves across as well as down the direction of vertical

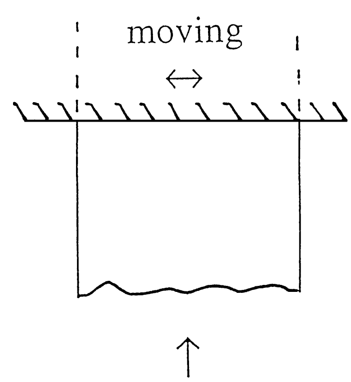

Figure 2 Cutting where the blade moves horizontally as well as vertically.

movement (or, equivalently, the knife 'slides' across and perpendicular to the straight line approach velocity of the workpiece). There is a force V normal to the cutting edge and a force H parallel to the cutting edge. The associated displacements are v and h respectively. We assume that the incremental work for cutting is always $[Rw\mathrm{d}v]$ but now some of that work will be provided by the sideways action of the blade.

In unit time, let V move through $\mathrm{d}v$ and H through $\mathrm{d}h$. The incremental work done is therefore $[V\mathrm{d}v + H\mathrm{d}h]$. In the absence of friction, this provides the fracture work required for the increment of new cut area, which is given by $Rw\mathrm{d}v$, assuming that the growth of cut keeps steady with the movement of the blade. Thus

$$V\mathrm{d}v + H\mathrm{d}h = Rw\mathrm{d}v$$

giving

$$(Rw - V)\mathrm{d}v = H\mathrm{d}h \tag{4}$$

When $\mathrm{d}h = 0$, we recover the orthogonal microtoming relation (1).

The resultant force is given by $[V^2 + H^2]^{1/2}$ and the resultant displacement is $[(\mathrm{d}v)^2 + (\mathrm{d}h)^2]^{1/2}$. When there is no permanent distortion of the offcut, these increments are *coincident in the plane of the cut* (it may be shown that in oblique metal cutting with chips permanently distorted out of the plane of the cut, the increments of force and displacement are *not* coincident and this simplification is absent [8]). We may therefore also write for the work increment in removing frictionless floppy offcuts

$$[V^2 + H^2]^{1/2}[(\mathrm{d}v)^2 + (\mathrm{d}h)^2]^{1/2} = Rw\mathrm{d}v \tag{5}$$

Calling the 'slice/push' ratio given by $\mathrm{d}h/\mathrm{d}v = \xi$, Equations 4 and 5 become

$$(Rw - V) = H\xi \tag{6}$$

$$[V^2 + H^2]^{1/2}[1 + (\xi)^2]^{1/2} = Rw \tag{7}$$

Replace V by $(Rw - H\xi)$ in (7) and manipulate to obtain

$$[H/Rw] = \xi/[1 + \xi^2] \tag{8}$$

Substitution back gives

$$[V/Rw] = 1/[1 + \xi^2] \tag{9}$$

The consequence of co-linearity between resultant force and resultant displacement in the plane of cut is that $H = \xi V$. The non-dimensional resultant force (F_{Res}/Rw) is

$$(F_{\mathrm{Res}}/Rw) = (1/[1 + \xi^2])^{1/2} \tag{10}$$

where ξ is the ratio of 'horizontal' to 'vertical' displacements or, more conveniently, the ratio of 'horizontal' to 'vertical' speeds. ξ is called the 'slice/push' ratio in industry.

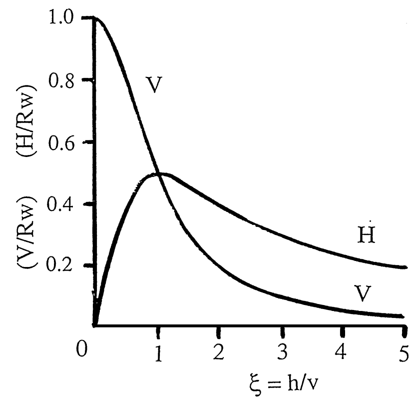

Figure 3 Variation of normalised forces for frictionless orthogonal cutting with 'slice/cut' speed ratio ξ. The vertical force V decreases as soon as sideways motion is introduced ($\xi > 0$). The horizontal force H increases at first but passes through a peak (at $\xi = 1$ in this frictionless case) where it has the same value as V and then diminishes. V diminishes at a faster rate and is then always $<H$. The resultant force diminishes from the outset.

The variation of H, V and F_{Res} with ξ is shown in Fig. 3. For $\xi = 0$, $H = 0$ and $V = Rw$. For $\xi \to 1$, H increases to a peak at $\xi = 1$ (when $H/Rw = V/Rw = 0.5$) and then diminishes as ξ increases. V diminishes for all ξ. Obviously smallest normalised forces occur for largest ξ, i.e., the sideways speed has to be as great as possible to reduce cutting forces so long as R is constant (we recognise that strain rate effects may very well affect R and we shall later see that the effect of friction suggests that there is no point in increasing ξ indefinitely).

The common experience of V diminishing quickly as soon as some sideways motion is introduced is immediately apparent from Fig. 3. The effect is noticeable because it is disproportionate: there is, in effect, a nonlinear coupling between V and H since vertical blade displacement and the area of new cut both depend upon v. Since a knife failing to penetrate with only a vertical force will be almost at rest, the slightest horizontal motion will cause $\xi > 0$ and hence much reduce V, as found practically.

Friction acts in the direction of the resultant displacement of the blade. The components of friction force are

$$\left.\begin{aligned} &2Lw\tau_{\mathrm{f}}[\xi/\surd(1 + \xi^2)] \\ &\quad \text{in the } h \text{ ('tangential') direction; and} \\ &2Lw\tau_{\mathrm{f}}[1/\surd(1 + \xi^2)] \\ &\quad \text{in the } v \text{ ('normal') direction.} \end{aligned}\right\} \tag{11}$$

The factor 2 is to represent the two sides of the cutting blade which are usually in contact with the offcut. From adding in the incremental friction work to the basic work Equations 4 and 5, it transpires that the X, Y and F_{res} forces in the presence of friction are given by the corresponding frictionless forces (Equations 8–10), but with a multiplying factor given by

$$[1 + M\surd(1 + \xi^2)] \tag{12}$$

where $M = (2L\tau_{\mathrm{f}}/R)$ as before in Equation 3. When $M = 0$, the equations revert to those for frictionless cutting.

Finite M has the effect of not only increasing V and H but also shifting them to the right, i.e., to higher ξ. H still peaks, but at greater ξ than in frictionless cases. After the peak, H does not fall off so rapidly and friction tends to keep H high, so that in practical cutting it is not advantageous to increase ξ beyond where H levels off.

3. Experiments

A wire band saw, or a disc ('bacon') slicer, suitably instrumented to pick up the force perpendicular to the blade (the 'feed' force) and the force along the direction of motion of the cutting edge, are suitable testbeds to assess the analysis. Such devices have the advantage of continuous motion in one direction of the cutting edge. (How the analysis may be employed for reciprocating cutting devices is discussed in Section 4 later). Experiments employing a disc cutter (delicatessan slicer) were recently performed at various temperatures and rates in a comprehensive study of the cutting of a wide variety of foodstuffs, full details of which are reported elsewhere [9]. The cuts were very thin. Under these conditions, the offcuts were not permanently deformed and order-of-magnitude calculations show that stored elastic energy was negligible. Consequently the assumption of the analysis that there is negligible energy (elastic or plastic) in the offcut is justified.

For present purposes we illustrate results for just two materials, viz: commercial cheddar cheese and pepper salami, both cut at between $-6°$ and $-10°$C. (Our wheel cutting device is not fitted with a temperature-controlled cabinet, so the temperature of the cheese during cutting is a little uncertain. However, we have performed extensive independent mechanical property measurements on the same materials over a wide range of rates in a temperature-controlled cabinet, and there are no sudden transitions in properties over the band of uncertainty in temperature of the cutting device). Cheddar cheese has been chosen to illustrate the applicability of the theory because not only do biological materials display far more scatter than engineering materials, but also the cutting of cheese is notoriously affected by friction (hence the use of wire to cut cheese). Our device has 200 mm diameter wheel with a 5 mm long bevel to the sharp cutting edge. The slice/push ratio ξ is altered by keeping the feed rate fixed and altering the wheel speed. The feed rate employed was slow (ca. 5 mm/s).

Fig. 4 shows representative results for cutting very thin slices from a 45 mm thick block of cheddar cheese. We assume that we have no idea of appropriate values of R and τ_f and we find 'best fit' values which are subsequently compared with independently-determined material properties. To aid curve fitting, we note that the ξ value at which the normalised H force given by Equation 8 multiplied by Equation 12 passes through a maximum depends only on M. The most appropriate M value is that which produces a force maximum at the ξ value observed experimentally. Calculations are relatively insensitive to both ξ and M and are readily performed with a spreadsheet. (The theory gives the non-dimensionalised force, so implicit in this procedure is that R is constant and independent of ξ). We find that for $1.8 \sim \xi_{peak} \sim 2.0$, $1.2 \sim M \sim 1.7$.

Knowing M, the toughness value may be calculated either from the V force intercept at $\xi = 0$, or from the $H = V$ value at $\xi = 1$ which is given by

$$H = V = [Rw/2][1 + M\sqrt{2}]$$
$$= (1.3\text{–}1.7)Rw$$

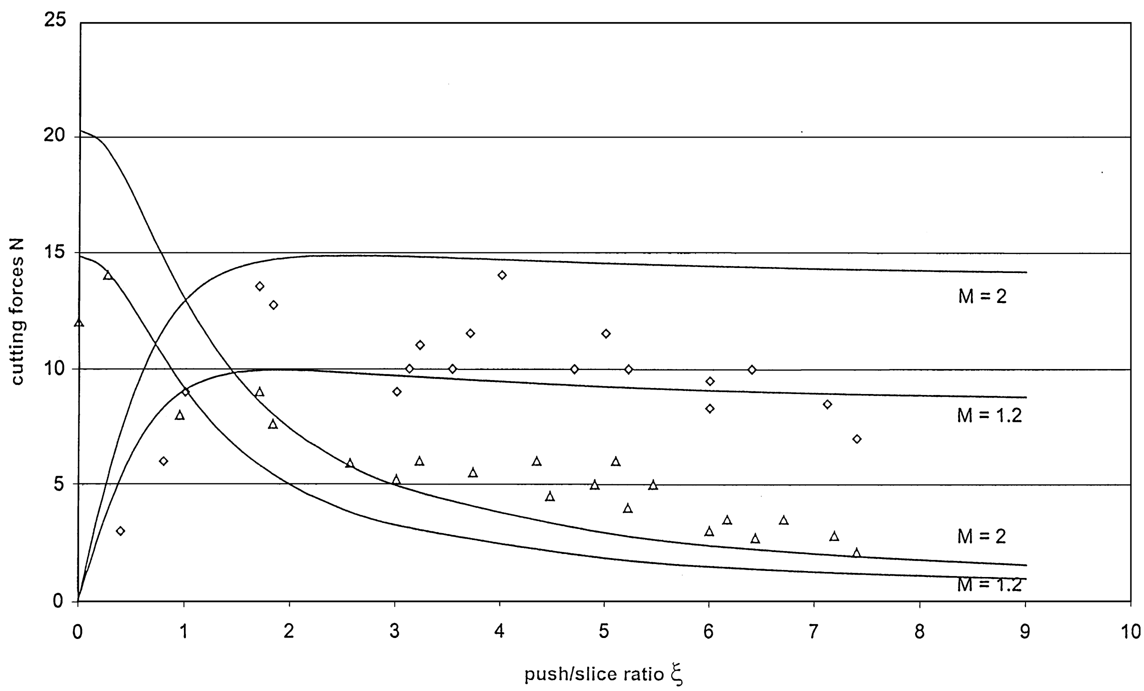

Figure 4 Results for wheel cutting of cheddar cheese at -6 to $-10°$C. Open triangles are the experimental 'feed' force into the wheel; open diamonds are the experimental wheel forces. The curves are the predictions of Equations 8–10 multiplied by Equation 12, with $M = 1.2$ and 2.0, from which $R = 120\text{–}150$ kJ/m^2 and $\tau_f = 20$ kPa ($\tau_y = 40$ kPa) which agree with independently-determined values of toughness and yield strength at low rates in the same temperature range.

for M in the range 1.2–1.7. For $H = V \approx 9$ N, Fig. 4, we find $120 \sim R \sim 150$ J/m^2. The predictions of the analysis—i.e., Equations 8 and 9 all multiplied by Equation 12—are superimposed in Fig. 5 for the friction parameter $M = 1.2$ and 1.7.

Independent estimates from our own work of the fracture toughness of the same cheese in the same temperature range, are some 50 to 200 J/m^2 from precracked 3-point bend testipieces, values depending on temperature and strain rate. The lower the temperature, and the greater the strain rate, the greater the toughness. The lowest cracktip strain rate in our bend tests was 0.25 s^{-1}; the highest was 4 s^{-1}. Our cutting experiments were performed at slow speeds which are at low strain rate. At room temperature we found that the toughness of the same cheese was much lower at some 5–10 J/m^2 agreeing with [10].

Inspection of the cutting wheel in operation suggests that the blade is in contact with the cheese over the sides of the edge bevel which extends for some 5 mm; cheese and blade separate thereafter. For $1.2 \sim M = [2L\tau_f/R] \sim 1.7$, use of $L = 5$ mm and the R found above gives $\tau_f \approx 20$ kPa. There was transfer of cheese to the cutting wheel so sticking friction seems likely. In consequence, if τ_f is identified with the shear yield stress ($\approx$yield strength/2), it follows that $\sigma_y \approx 40$ kPa. Our own measurements for yield strength in that temperature range are some 40–100 kPa, depending upon rate (higher for higher strain rates). Our slow cutting experiments correspond with the lower end of these strain rates. Although we found that σ_y for our cheddar cheese did change with rate at fixed temperature, it did not increase much until the temperature dropped below -15°C. Thus these σ_y are comparable to the room temperature values given by Kamyab *et al.* [10].

Fig. 5 gives the corresponding results for a commercial pepper salami. The 'wheel' cutting force peaks at $1.5 \sim \xi \sim 2.5$ and it is found that $0.75 \sim M \sim 2$, from which $70 \sim R \sim 100$ J/m^2 and $10 \sim \tau_f \sim 14$ kPa.

4. Discussion and conclusions

A theory, based upon work, has been presented which explains why the forces required for cutting by 'pressing down and simultaneously sliding sideways' are much less that the force for cutting by 'pressing down alone'. The predictions of the theory accord with common experience, and experiments using instrumented cutters support the trends of the analysis (see also [9] for other foodstuffs). Figs 4 and 5 show that the analysis predicts the experimental behaviour reasonably well, except that the 'feed' force is underestimated at larger slice/push ratios. It is believed this is down to the method of modelling friction. Despite the somewhat scattered results for cutting, and that the fits to the theory in Figs 4 and 5 are consequently not very 'tight', the derived values of fracture toughness and yield strength agree with our own independent data and those of others (e.g., [10]). Scatter in results is, however, typical of biological materials, which rarely are as well behaved as traditional engineering materials.

The analysis is presented in terms of continuous sideways motion of the blade, as in a band wire or circular cutting disc. However it is readily adapted to cutting with a reciprocating blade. In such cases the blade velocity is zero at either end of the stroke and passes through a maximum at mid-stroke. If driven by a crank mechanism, the velocity can be represented approximately by a sine curve. It follows that for constant downwards velocity, the ratio of sideways to vertical velocities, given by ξ, is continuously changing also according to the sine function. The variation in both vertical and horizontal forces throughout one stroke may be found by calculating the values at the changing ξ [11]. Unlike continuous cutting in one direction where

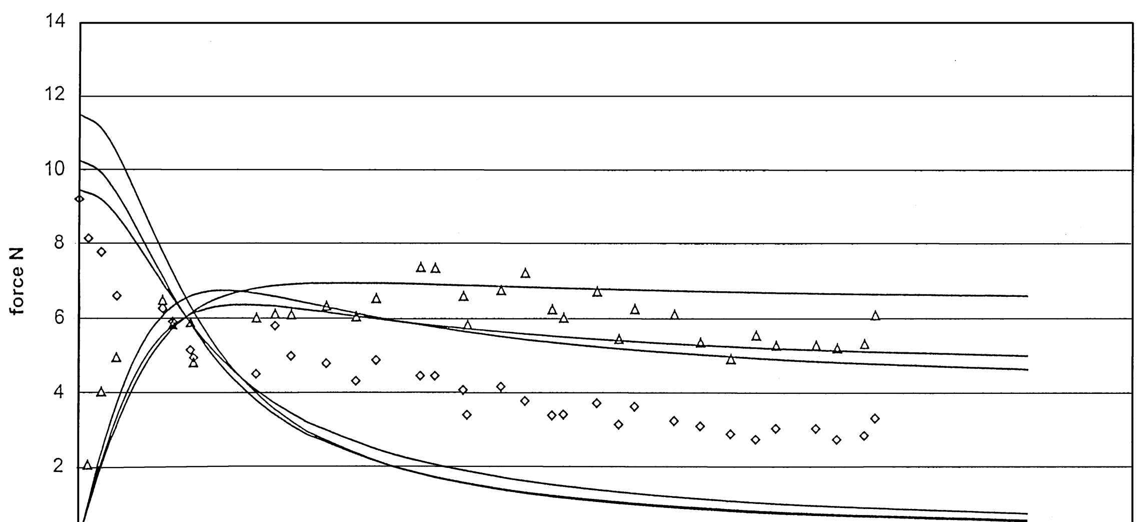

Figure 5 Results for wheel cutting of pepper salami at –6 to –10°C. Open diamonds are the experimental 'feed' force into the wheel; open triangles are the experimental wheel forces. The curves are the predictions of Equations 8–10 multiplied by Equation 12, with $M = 0.5$, 0.75 and 2, from which $R = 70$–100 kJ/m^2 and $\tau_f = 10$–14 kPa ($\tau_y = 20$–30 kPa).

ξ is maintained at a high value in order to reduce the cutting forces, reciprocating devices experience a range of ξ from zero (at both ends of the stroke) up to that given by the fastest speed at mid-stroke. It follows that a *range* of forces, rather than steady forces, must be encountered when reciprocating blades are employed. Of note, the range must include the high forces at the ends of the reciprocating stroke (where ξ is small), irrespective of how fast the blade goes in between. Force signals from reciprocating devices are therefore very 'spikey' and the benefits of high slice/push ratios in continuous cutting are not fully obtained. As part of the wide-ranging LINK programme reported by Atkins *et al.* [9], an instrumented jigsaw was mounted within the frame of a uniaxial testing machine and specimens attached below the crosshead, thus permitting the measurement of V and H under a wide range of ξ. The analysis has also been applied to commercial food cutting machines having orbital and involute (spiral) blades.

In this paper the theory has been limited to cases where the offcut is very thin and thus floppy, and therefore stores no elastic energy. The analysis may, however, be extended to the situation where the offcut is permanently deformed either by bending, or by shear, and will be presented elsewhere. Permanent bending is found in the free tearing of paper: permanent twisting of offcuts into regular helices occurs when thin slivers of paper are guillotined off the edges of bigger sheets. A (rigid-plastic) explanation for simultaneous plastic bending and plastic twisting of offcuts in the guillotining of ductile metal sheets and plates may be found in Ref. [12]. Permanent deformation of the offcut by shear is found in metalcutting operations. A rigid-plastic version of the theory has recently been applied to describe the mechanics of metal cutting [13], where the textbook wisdom that machining concerns only plasticity and friction is challenged. That is, it is argued that the work of machining consists not only of the plastic work of chip formation and the work against friction, but also must include fracture work at the 'ductile fracture mechanics' levels of some 10's or 100's of kJ/m^2 [14]. The new analysis explains a number of things that the traditional theory is incapable of explaining, such as the material dependence of the primary shear plane angle, the positive intercepts always found in diagrams of cutting force vs. uncut chip thickness (but which are ignored in data reduction) and the so-called 'scale effect' in cutting. It also explains why modern finite element models (FEM) of metalcutting have to employ a 'separation criterion' at the tool tip, in addition to the usual material properties, in order to enable the cutting tool to move. The cutting behaviour of ductile materials depends on their 'toughness/strength' ratios, not only on their 'strength', and thermomechanical treatments alter such ratios, i.e., a metal with the same hardness in different conditions may have quite different toughnessess and hence quite different machining behaviour.

In the case of oblique cutting, whatever the behavior of the offcut, the sense of direction of the sideways motion becomes important, from which it may be shown that cutting 'downhill' is easier than cutting 'uphill'.

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